where **ON** = vector in direction of **H** of magnitude R_e, **OP** = vector in Z direction of magnitude R_e. Then

$$\cos PN = H_z \tag{28}$$

In a similar fashion

$$\cos NS' = \mathbf{RN}\cdot\mathbf{RS}/|\mathbf{RS}||\mathbf{RN}| = \mathbf{H}\cdot\mathbf{RS}/|\mathbf{RS}| \tag{29}$$

and

$$\cos S'P = \mathbf{RS}\cdot\mathbf{OP}/|\mathbf{RS}||\mathbf{OP}| = RS_z/|\mathbf{RS}| \tag{30}$$

Using spherical trigonometry

$$\cos PNS' = \frac{\cos S'P - \cos PN \cos NS'}{\sin PN \sin NS'} \tag{31}$$

$$\cos\zeta = \frac{RS_z - H_z(\mathbf{RS}\cdot\mathbf{H})}{[\{|\mathbf{RS}|^2 - (\mathbf{RS}\cdot\mathbf{H})^2\}(1 - H_z^2)]^{1/2}} \tag{32}$$

$$\tan\zeta = \frac{RS_xH_y - RS_yH_x}{RS_z(H_x^2 + H_y^2) - H_z(RS_xH_x + RS_yH_y)} \tag{33}$$

Applications

A FORTRAN program[1] for the Control Data 3100 was written to perform the preceding calculations and output day by day tables of rise time, set time, time of closest approach, and elevation and azimuth at time of closest approach for the NNSS satellites. These tables have been used successfully to predict and identify satellite passes.

The long-range accuracy of prediction is of course based upon the stability of the broadcast precession parameters and the accuracy of the other orbit parameters. Some study was made of the drift over several months. Alert calculations for four days in early March, 1969, were carried out using one set of satellite parameters from March 1969 and one from early June 1968. The rise (and set) times were very consistently 10 min earlier with the March data. The azimuth at t_{ca} was 2° less (average referred to the March data) with a standard deviation of 7°.

The average change in elevation was also 2° with a standard deviation of 7°. The elevation change was greater at greater elevations, thus there were only 5 passes of the 140 predicted in the four days which were not common to both sets of alerts.

It would thus seem that alerts can be predicted for a long time in advance, at least nine months in our experience, and that the rise and set times if corrected by a fixed constant of about −1 min per month, can be used to identify satellite passes.

Reference

[1] Budlong, K. S., "Fortran Satellite Alert Programs for the Control Data 3100," BI Computer Note 68-8-C, Oct. 1968, Bedford Institute, Dartmouth, N.S., Canada.

Temperature Predictions during Spacecraft Maneuvers

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Nomenclature

A_c = projected area of the component whose temperature response is desired
A_m = is the measured area of component 1 in the sketch made with a viewer
A_1 = total surface area of component 1
A_1^{P2} = projected area of component 1 to 2
$A_1^{P2'}$ = projected area of component 1, as seen by the illuminated portion of component 2
A_1^{PS} = projected area of component 1 toward the sun
C_1,C_2 = $\psi\epsilon_1A_1\sigma$ and mc_p, respectively, constants in Eq. (6)
c_p = specific heat of component 1
$F_{1\text{-}2}$ = view factor from 1 to 2
$F_{1\text{-}2'}$ = view factor from component 1 to illuminated portion of 2
$F_{1\text{-}S}$ = "effective" view factor to space. This portion of the energy that leaves 1 does not return (includes energy which bounces off of 2 to space)
L_c = is a component length (see Fig. 3)
L_m = measured length in the sketch corresponding to a known length on a component
m = mass of component 1
Q_G = internal heat generation
Q_S = $\alpha_1A_1^{PS}S$ = direct solar heat input
Q_R = $\epsilon A_1\sigma T_1^4$ = radiated heat from component 1
Q_{RS} = $\rho_2\alpha_1^*A_1^{F2'}SF_{1\text{-}2'}$ = reflected solar heat input
Q_{IR} = $\epsilon_1^2\epsilon_1^2A_1^{P2}\sigma F_{1\text{-}2}T_2^4$ = infrared heat input from surroundings
Q_{stored} = $mc_pdT/d\theta$ = thermal storage of component 1
S = solar constant
T = absolute temperature; T_i = initial value of T_1; T_2 = effective T of component 2
α_1,α_1^* = effective absorptances of 1 to direct from sun and to reflected energy from 2; it is assumed that $\alpha_1^* = \alpha_1$
ϵ_1 = effective emissivity of component 1
$\epsilon_1^2,\epsilon_2^1$ = emissivity of component 1 as seen by component 2, and vice versa
σ = Stephan-Boltzmann constant
ψ = factor defined by Eq. (3)
θ = time

Subscripts

1 = component whose $T(\theta)$ is to be calculated
2 = rest of spacecraft

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Introduction

During midcourse or terminal maneuvers, the spacecraft will assume a different position with respect to the sun. Since this position is not known prior to flight, exact thermal predictions cannot be made beforehand. In the past, only worst-case predictions (total eclipse or the worst-case solar heating) were relied upon to determine if a maneuver was acceptable. This is an acceptable answer only if temperature limits are not exceeded. If temperature limits are exceeded with a worst-case analysis, then more accurate predictions are needed. This Note presents a method for quickly but accurately calculating the temperature response of spacecraft components in a nonstandard solar orientation.

Governing Equation

The equation describing the temperature history of a spacecraft component is

$$Q_S + Q_{RS} + Q_{IR} + Q_G = Q_R + Q_{\text{stored}} \tag{1}$$

Equation (1) can be rewritten so that the direct solar heat input is separate from the other heat input term as follows:

$$\alpha_1A_1^{PS}S - \psi A_1T_1^4 = mc_p(dT/d\theta) \tag{2}$$

Here ψ is a correction factor that incorporates the infrared heat input from the surroundings as well as the reflected solar heat input and internal heat generation. More specifically, ψ is equivalent to

$$\psi = F_{1-S} - (1/\epsilon_1A_1\sigma T_1^4)[\epsilon_1^2\epsilon_2^2A_1^{P2}F_{1-2}\sigma T_2^4 + \rho_2\alpha_1^*F_{1-2}'A_1^{P2'}S + Q_G] \tag{3}$$

It will be noted that ψ is a constant if the following realistic conditions are met:

1) The temperature response of the surroundings (2) are about the same as the response of the component of interest (1) and that T_1 and T_2 are not greatly different.

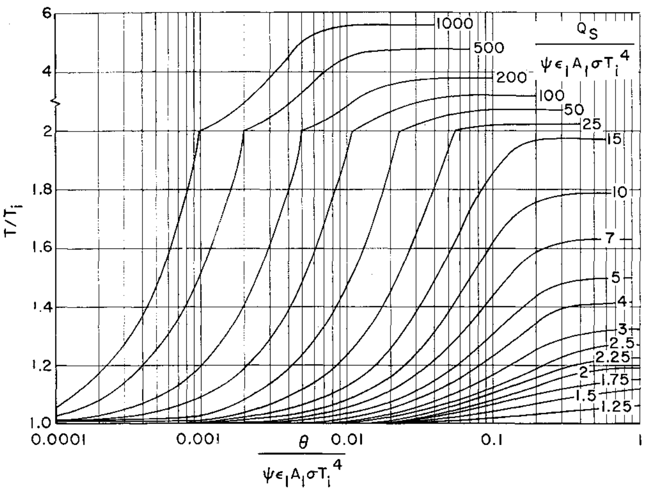

Fig. 1 Heating history of radiating isothermal solid.

2) The reflected solar input does not change during the maneuver, or the reflected solar input is small with respect to the other terms of Eq. (3). If this is not the case ψ cannot include the reflected solar input and changes in reflected solar input will have to be evaluated as well as the direct solar heat input change.

The constant correction factor ψ can be obtained from solar thermal vacuum test data of the spacecraft. Either normal orientation steady-state or eclipse data can be used to evaluate ψ. The steady-state and eclipse equations are

$$\psi\epsilon_1 A_1 \sigma T_1{}^4 = Q_S \tag{4}$$

$$\psi\epsilon_1 \sigma T_1{}^4 = -mc_p dT_1/d\theta \tag{5}$$

In addition to the aforementioned conditions, one also must be aware that Eqs. (1) and (2) assume the component to be isothermal (or all one temperature). Conduction effects to the surrounding components also are neglected. These assumptions will have to be tolerated if quick predictions are to be made. As a matter of interest, the isothermal assumption does not seem to be too restrictive if bulk system behavior is desired.

Upon examination of Eq. (2) it can be seen that it has the following form:

$$C_1 T_1{}^4 + C_2(dT/d\theta) = Q_S \tag{6}$$

Equation (6) has been solved,[1,2] and the solution is presented in dimensionless form in Figs. 1 and 2.

Prediction Procedures

Once the new spacecraft orientation with respect to the sun is known, the change in solar heat input can be calculated. To do this quickly and accurately, one can use a viewer to get the new projected area toward the sun (Fig. 3). The viewer could be one of several types. A partially silvered mirror, a beamsplitter, or a simple device consisting of a mirror, a plain piece of glass, and a housing could be used. Besides the devices shown in Fig. 3, there are commercially available devices such as the camera Lucida or the camera Obscura (opaque projector) which could be used for this use. Photographic techniques could also be used to achieve the desired projected area.

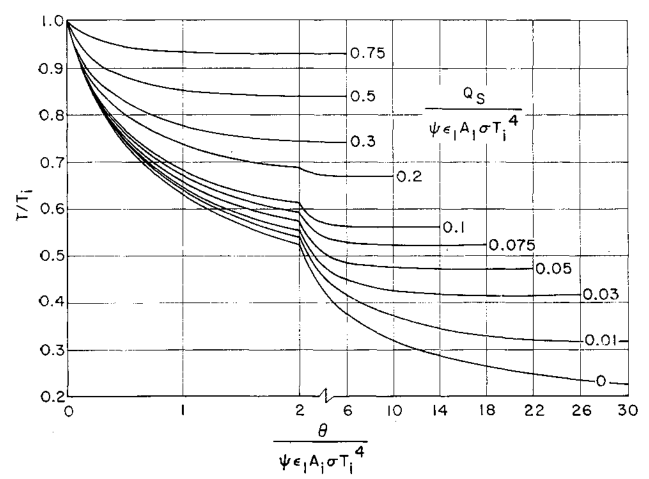

Fig. 2 Cooling history of radiating isothermal solid.

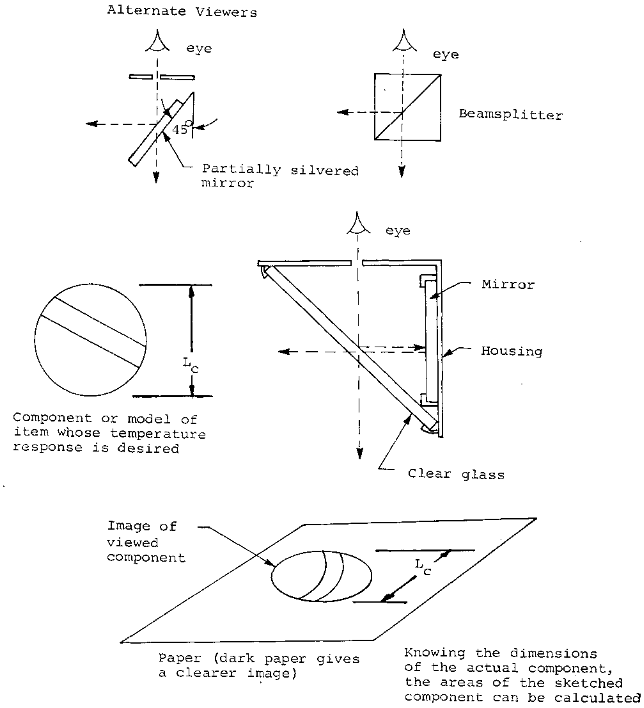

Fig. 3 Method of obtaining projected areas.

The sketched projected areas can be measured using a planimeter or by overlaying clear graph paper and counting squares. Care must be taken to scale the measured area A_m to the actual area A_c using the following relationship:

$$A_c = A_m(L_c/L_m)^2 \tag{7}$$

In order to get the changed solar input Q_S, a scale model of the spacecraft or individual components (correctly painted) are needed. Individual components are desired as they are larger than they would be on a model, and the resulting sketch would also be larger and more accurate. The scale model is needed if components are shadowed from the sun by other parts of the spacecraft. The dimensions of the component are needed for Eq. (7). Once the A_c's of the various thermal coatings of the component are known, the revised Q_S is obtained by multiplying projected areas by the absorptance and the solar constant, and summing these individual $\alpha A_c S$ values.

Knowing the new value of Q_S for the maneuver orientation, it is a simple matter to calculate the dimensionless parameters that are required in order to utilize the curves presented in Figs. 1 and 2, which describe the thermal response of a radiating isothermal solid with a heat input.

References

[1] Ishimoto, T. and Randolph, B. W., "The Prediction of Transient Thermal Environments in Space Vehicles," TM 650, April 1960, Hughes Aircraft Co., Culver City, Calif.

[2] Ohanesian, P. J., Jr., "Non-Dimensional Tables for Transient Response of a Radiating One-Node System to a Constant Heat Input," TXAID 179, Sept. 1967, Lockheed Missiles & Space Co., Sunnyvale, Calif.